**Name and Surname** : ..........................................................................................

**Grade/Class** : 11/......... **Mathematics Teacher** : ..............................

GRADE 11
MATHEMATICS

JUNE EXAMINATION
2023

**ANSWER BOOK**

| |
|---|
| |
| 150 |

QUESTION 1

| | | |
|---|---|---|
| 1.1.1. | $2x^2 - 13x + 15 = 0$ | |
| | $(x-5)(2x-3) = 0$ ✓ | |
| | $x = 5$ or $\frac{3}{2}$ ✓ | 2 |
| | | |
| | | |
| 1.1.2. | $5x^2 - 2x - 8 = 0$ | |
| | | |
| | $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(5)(-8)}}{2(5)}$ ✓ | |
| | | |
| | $= \frac{2 \pm \sqrt{164}}{10}$ | |
| | | |
| | $= 1{,}48$ ✓ or $-1{,}08$ ✓ | 3 |
| | | |

1.1.3. (a) $6 \leq x^2 + x$

$0 \leq x^2 + x - 6$

$0 \leq (x-2)(x+3)$

$+ \quad 0 \quad - \quad 0 \quad +$

$-3 \qquad 2$

$x \leq -3$ or $2 \leq x$ or 0 — 3

(b) $x^3 + x^2 \leq 0$

$x^2(x+1) \leq 0$

$\ominus \quad 0 \quad + \quad 0 \quad +$

$-1 \qquad 0$

$x \leq -1$ or $x = 0$ — 3

1.1.4. $\sqrt{x+5} - 3 = x$

$\left(\sqrt{x+5}\right)^2 = (x+3)^2$

$x + 5 = x^2 + 6x + 9$

$0 = x^2 + 5x + 4$

$0 = (x+1)(x+4)$

$\therefore x = -1$ or $\cancel{-4}$ reject — 4

a+5

| | | |
|---|---|---|
| 1.1.5. | $(3x^{\frac{4}{3}}-5)(3x^{\frac{3}{5}}+4)=0$ | |
| | $x^{\frac{4}{3}}=\frac{5}{3}$ or ✓ $x^{\frac{3}{5}}=-\frac{4}{3}$ | |
| | $x=\pm\left(\frac{5}{3}\right)^{3/4}$ method ✓ $x=\left(-\frac{4}{3}\right)^{5/3}$ | |
| | $x=\pm 1,47$ $\quad x=-1,62$ | 5 |
| | ✓ ✓ | |
| | | |
| 1.1.6. | $2.3^{2x}-3^{x}-6=0$ | |
| | $(3^{x}-2)(2.3^{x}+3)=0$ ✓ | |
| | $3^{x}=2$ or $3^{x}=-3/2$ ✓ both | |
| | no soln ✓ | |
| | $x=\frac{\log 2}{\log 3}$ ✓ | |
| | $=0,63$ ✓ | 5 |
| | | |
| | | |
| 1.2.1 | $2y-x=-3 \quad \therefore 2y+3=x$ ✓ | |
| | $x^{2}-3xy+y^{2}-2x+7y=11$ | |
| | $(2y+3)^{2}-3(2y+3)y+y^{2}-2(2y+3)+7y=11$ ✓ | |
| | $4y^{2}+12y+9-6y^{2}-9y+y^{2}-4y-6+7y-11=0$ | |
| | | |
| | $-y^{2}+6y-8=0$ | |
| | $\div -1$: $y^{2}-6y+8=0$ ✓ | |
| | $(y-2)(y-4)=0$ ✓ | |
| | $\therefore y=2$ or $4$ ✓ both | |
| | $\therefore x=2(2)+3$ or $2(4)+3$ | |
| | $=7$ $\quad =11$ ✓ both | 6 |
| | | |

| | | |
|---|---|---|
| 1.2.2. | $\frac{3^{y+1}}{32} = \sqrt{96^x}$ | |
| | $\frac{3^{y+1}}{2^5} = 96^{\frac{x}{2}}$ | |
| | ✓LHS $= (3.2^5)^{x/2}$ | |
| | $3^{y+1}.2^{-5} = 3^{x/2}.2^{5x/2}$ ✓RHS | |
| | $2: \ -5 = \frac{5x}{2}$     $3: \ y+1 = \frac{-2}{2}$ | |
| | $-2 = x$ ✓     $y = -2$ ✓ | 4 |
| 1.3. | $2.5^n - 5^{n+1} + 5^{n+2}$ | |
| | $= 2.5^n - 5^n.5^1 + 5^n 5^2$ | |
| | $= 5^n(2 - 5 + 5^2)$ ✓ | |
| | $= 5^n.22$ ✓ | |
| | $= 5^n.11.2$ | |
| | $= \underbrace{(5^n.11)}\times 2$ ✓ $\ldots \times 2$ | |
| | $\in \mathbb{N}$ if $n \in \mathbb{Z}^+$ | |
| | $\therefore$ even ✓ | 4 |

QUESTION 2

| | | |
|---|---|---|
| 2.1.1. | $(1+\sqrt{2}-\sqrt{18})(1+2\sqrt{2})$ | |
| | $\cdot\ \sqrt{18} = \sqrt{9.2} = 3\sqrt{2}$ ✓ 9.2 | |
| | $(1+\sqrt{2}-3\sqrt{2})(1+2\sqrt{2})$ | |
| | $= (1-2\sqrt{2})(1+2\sqrt{2})$ | |
| | $= 1 - 4.2$ | |
| | $= -7$ ✓ | 3 |
| | | |
| | | |
| 2.1.2. | $\frac{8}{\sqrt{2}-2} \times \frac{\sqrt{2}+2}{\sqrt{2}+2}$ ✓ $= \frac{8\sqrt{2}+16}{2-4}$ | |
| | $= \frac{8\sqrt{2}+16}{-2}$ ✓ ✓ | |
| | $= \frac{8\sqrt{2}}{-2} - \frac{16}{2}$ | |
| | $= -4\sqrt{2} - 8$ ✓ | 4 |
| | | |
| | | |
| 2.2. | $9^{x-1} = (3^2)^{x-1}$ | |
| | $= 3^{2x-2}$ ✓ | |
| | $= 3^{2x} . 3^{-2}$ | |
| | $= (3^x)^2 . \frac{1}{3^2}$ | |
| | $5^2$ ✓ $= (5)^2 . \frac{1}{9}$ | |
| | $= \frac{25}{9}$ ✓ | 3 |
| | | |
| | | |
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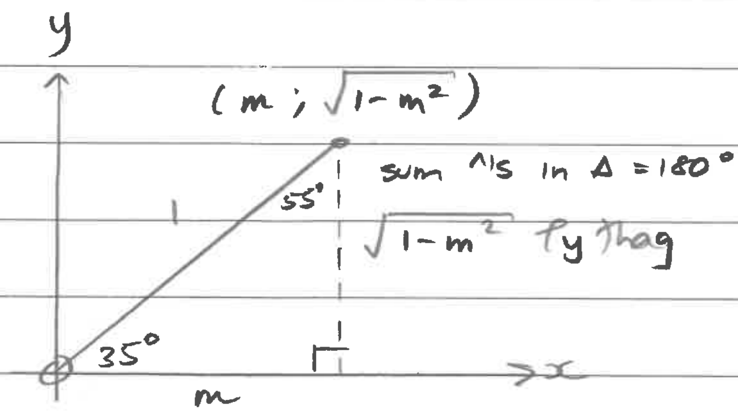
| | | |

QUESTION 3

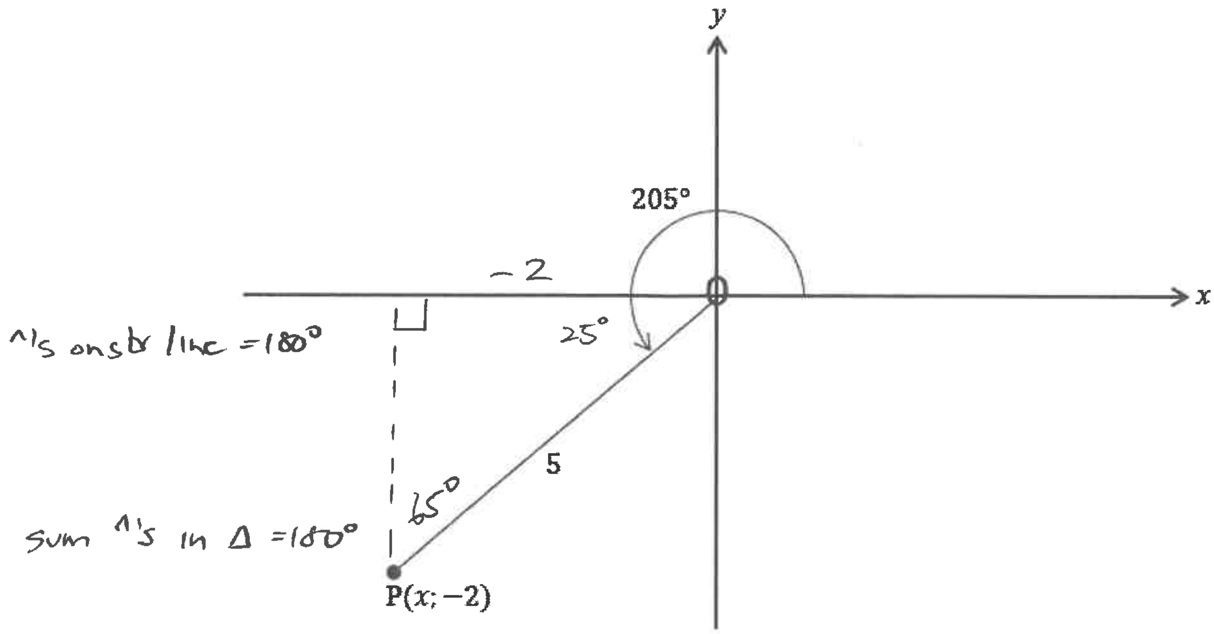
| | | |
|---|---|---|
| 3.1. | $(x-5)(x-5)=0$ | |
| | $x^2-10x+25=0$ | |
| | $p=-10 \quad q=25$ | 4 |
| | | |
| 3.2. | $f: \quad y=3x+k \qquad g: y=x^2-2$ | |
| | $3x+k=x^2-2$ | |
| | $0=x^2-3x-k-2$ | |
| | $\Delta=(-3)^2-4(1)(-k-2)$ | |
| | $=9+4k+8$ | |
| | $=4k+17$ | |
| | No $\cap$ : $\Delta<0$ | |
| | $4k+17<0$ | |
| | $k<-\frac{17}{4}$ | 6 |
| | | |
| | | |
| 3.3. | $x(2ax-1)=2a+1$ | |
| | $2ax^2-x-2a-1=0$ | |
| | $\Delta=(-1)^2-4(2a)(-2a-1)$ | |
| | $=1-4(-4a^2-2a)$ | |
| | $=1+16a^2+8a$ | |
| | $=16a^2+8a+1$ | |
| | $=(4a+1)(4a+1)$ | |
| | $=(4a+1)^2$ | |
| | | |

If $a \in \mathbb{Q}$, then $\Delta$ = perfect square $\therefore$ roots are rational. 4

QUESTION 4

| | | |
|---|---|---|
| 4.1 | $\cos 35^\circ = m$ $\quad \frac{m}{1}$ $\quad \frac{x}{r}$ | |
| |  | |
| 4.1.1. | $\cos 215^\circ = \cos(180^\circ + 35^\circ)$ | |
| | $= -\cos 35^\circ$ ✓ | |
| | $= -m$ ✓ | 2 |
| 4.1.2. | $\tan 55^\circ = \frac{m}{\sqrt{1-m^2}}$ $\quad \frac{o}{a}$ $\quad$ x, r ✓ $\quad$ y ✓ $\quad$ ✓ans | 3 |

4.2.




| | | |
|---|---|---|
| 4.2.1. | $x^2 + (-2)^2 = (5)^2$ Pythag | |
| | $x^2 = 21$ | |
| | $x = \pm\sqrt{21}$ reject + | |
| | $= -\sqrt{21}$ ✓ | 1 |
| | | |
| | | |
| 4.2.2. | (a) $\sin 385°$ | |
| | $= \sin(180° + 205°)$ OR $\sin 25°$ ✓ | |
| | $= -\sin 205°$ ✓ $= \frac{2}{5}$ ✓ $\frac{a}{h}$ | |
| | $= -\left(\frac{-2}{5}\right)$ $\frac{y}{r}$ | |
| | $= \frac{2}{5}$ ✓ NB o, a, h sides of Δ ALWAYS + | 2 |
| | | |
| | (b) $\cos 65°$ | |
| | $= \frac{2}{5}$ ✓✓ $\frac{a}{h}$ NB + | 2 |
| | or 0 | |

4.3.1. $\dfrac{\cos 111^\circ}{\sin 159^\circ}$

$\cdot\ \cos 111^\circ = \cos(180^\circ - 69^\circ)$

$= -\cos 69^\circ$ OR $-\sin 21^\circ$

$\cdot\ \sin 159^\circ = \sin(90^\circ + 69^\circ)$

$= \cos 69^\circ$ OR $\sin 21^\circ$

$\therefore \dfrac{-\cos 69^\circ}{\cos 69^\circ} = -1$ ✓ ✓ ✓

3

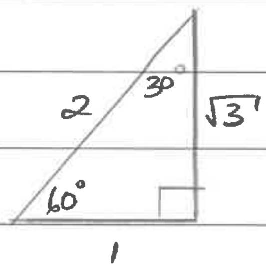
4.3.2. $\tan^2 330^\circ$

$= [\tan 330^\circ]^2$

$= [\tan(360^\circ - 30^\circ)]^2$

$= [-\tan 30^\circ]^2$ ✓

$= \left[-\dfrac{1}{\sqrt{3}}\right]^2$ ✓ $\dfrac{o}{a}$




$= \dfrac{1}{3}$ ✓

3

no spec diagram

max 2/3

QUESTION 5

| | | |
|---|---|---|
| 5.1. | LHS $\qquad$ RHS | |
| | $= \frac{1}{1-\sin x} - \frac{1}{1+\sin x}$ $\qquad = \frac{2\tan x}{\cos x}$ | |
| ✓ | $= \frac{1(1+\sin x) - (1-\sin x)}{(1-\sin x)(1+\sin x)}$ $\qquad = \frac{\frac{2\sin x}{\cos x}}{\frac{\cos x}{1}}$ ✓ num | |
| | $= \frac{1+\sin x - 1 + \sin x}{1-\sin^2 x}$ ✓ $\qquad = \frac{2\sin x}{\cos x} \times \frac{1}{\cos x}$ | |
| | $= \frac{2\sin x}{\cos^2 x}$ ✓ ✓ $\qquad = \frac{2\sin}{\cos^2 x}$ | 5 |
| | $\therefore$ LHS = RHS | |
| 5.2. | $9\sin^2 x - 7\sin x\cos x - 3$ | |
| | $= 9\sin^2 x - 7\sin x\cos x - 3.1$ | |
| | $= 9\sin^2 x - 7\sin x\cos x - 3(\sin^2 x + \cos^2 x)$ ✓ | |
| | $= 9\sin^2 x - 7\sin x\cos x - 3\sin^2 x - 3\cos^2 x$ | |
| | $= 6\sin^2 x - 7\sin x\cos x - 3\cos^2 x$ ✓ | |
| | $= (2\sin x - 3\cos x)(3\sin x + \cos x)$ ✓ | 3 |
| 5.3. | $\frac{\sin(-x-1710^\circ) - \cos(180^\circ + x)}{\cos(-x) - 3\sin(270^\circ + x)}$ | |
| | $\cdot \sin(-x-1710^\circ) = \sin(-x+90^\circ)$ | |
| | $= \sin(90^\circ - x)$ | |
| | $= \cos x$ | |
| | $\cdot \cos(180^\circ + x) = -\cos x$ | |
| | $\cdot \cos(-x) = \cos x$ | |

$\cdot \ \sin(270° + x) = -\cos x$

**NB:**

① Show reduction in brackets

② Then, simplify signs

$\therefore \dfrac{\cos x - (-\cos x)}{\cos x - 3(-\cos x)}$ ✓ ✓ ✓ ✓

$= \dfrac{\cos x + \cos x}{\cos x + 3\cos x}$

$= \dfrac{2\cos x}{4\cos x}$

$= \dfrac{1}{2}$ ✓

5

5.4.1. $2\cos x + \sqrt{3} = 0$

$\cos x = -\frac{\sqrt{3}}{2}$

ref ∠ $= 30°$

Cos − ive

$\text{II}: \ x = 150° + k.360°;\ k \in \mathbb{Z}$ ✓

or

$\text{III}: \ x = 210° + k.360°;\ k \in \mathbb{Z}$ ✓

2

5.4.2. $x \in [-360°; 180°]$

$x;\ -210°;\ 150°;\ x$

$x;\ -150°;\ \cancel{210}°$

$\therefore \ x = -210°$ or $\pm 150°$ ✓ all 3

1

| | | |
|---|---|---|
| 5.5.1. | $\cos x = 1$ | |
| | $x = k.360°;\ k \in \mathbb{Z}$ ✓ | 1 |
| | | |
| | | |
| 5.5.2. | $\sin 4x = 0$ | |
| | $\sin A = 0$ $\quad A = 4x$ | |
| | $A = k.180°$ | |
| | $4x = k\ 180°$ ✓ | |
| | $x = k\ 45°;\ k \in \mathbb{Z}$ ✓ | 2 |
| | | |
| | | |
| 5.5.3. | $3\sin x - 4\cos x = 0$ | |
| | $\div \cos x:\ \frac{3\sin x}{\cos x} - \frac{4\cos x}{\cos x} = \frac{0}{\cos x}$ | |
| | $3\tan x - 4 = 0$ ✓ | |
| | $\tan x = \frac{4}{3}$ ✓ | |
| | ref $\hat{}$ $= 53,13...°$ | |
| | tan is + in | |
| | $I:\ x = 53,13° + k.180°;\ k \in \mathbb{Z}$ ✓ | 3 |

KWT via Joburg : for 5.5.1 and 5.5.2.

5.5.1 $\cos x = 1$

ref $\angle = 0°$

Cos + ive

I : $x = 0° + k\,360°$ ; $k \in \mathbb{Z}$

or

IV : $x = 360° + k\,360°$ ; $k \in \mathbb{Z}$

5.5.2 $\sin 4x = 0$ $A = 4x$

$\sin A = 0$

ref $\angle = 0°$

Sin ± ive $0 = \pm$

I : $A = 0° + k\,360°$

$4x =$

$x = 0° + k.90°$ ; $k \in \mathbb{Z}$

II : $A = 180° + k.360°$

$4x =$

$x = 45° + k.90°$ ; $k \in \mathbb{Z}$

III : $A = 180° + k.360°$

same as II

IV : $A = 360° + k.360°$

$4x =$

$x = 90° + k\,90°$ ; $k \in \mathbb{Z}$

5.5.4. $\sin 2x + \cos(x - 10^\circ) = 0$

$A = 2x \qquad B = x - 10^\circ$

$\sin A + \cos B = 0$

$\sin A = -\cos B$

$\sin(270^\circ - B)$ III $\qquad\qquad$ $\sin(270^\circ + B)$ IV

$\sin A = \sin(270^\circ - B)$ or $\sin A = \sin(270^\circ + B)$

$A = 270^\circ - B + k.360^\circ \qquad A = 270^\circ + B + k.360^\circ$

$2x = 270^\circ - (x - 10^\circ) + k360^\circ \qquad 2x = 270^\circ + x - 10^\circ + k.360^\circ$

$2x = 270^\circ - x + 10^\circ + k.360^\circ \qquad x = 260^\circ + k.360^\circ; k \in \mathbb{Z}$

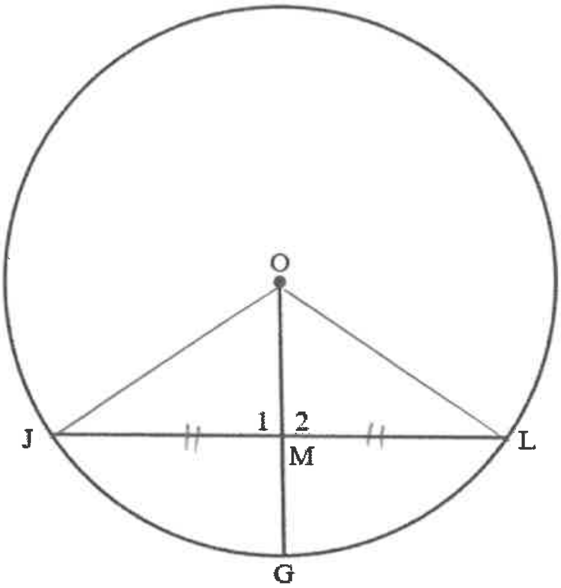
$3x = 280^\circ + k.360^\circ$

$x = 93{,}33^\circ + k.120^\circ; k \in \mathbb{Z}$

4

QUESTION 6

6.1.




✓ constr

| | | |
|---|---|---|
| | In $\Delta$'s OJM, OJL | |
| | 1. $OM = OM$ common | |
| | 2. $OJ = OL$ radii | |
| | 3. $JM = ML$ given } all 3 ✓ | |
| | $\therefore \Delta OJM \equiv \Delta OJL$ ✓S SSS ✓R | |
| | $\therefore \hat{M}_1 = \hat{M}_2$ ✓SR $\Delta OJM \equiv \Delta OJL$ | |
| | $\therefore \hat{M}_1 = \hat{M}_2 = 90°$ ✓SR ∠'s on str line $= 180°$ | 6 |
| | | |
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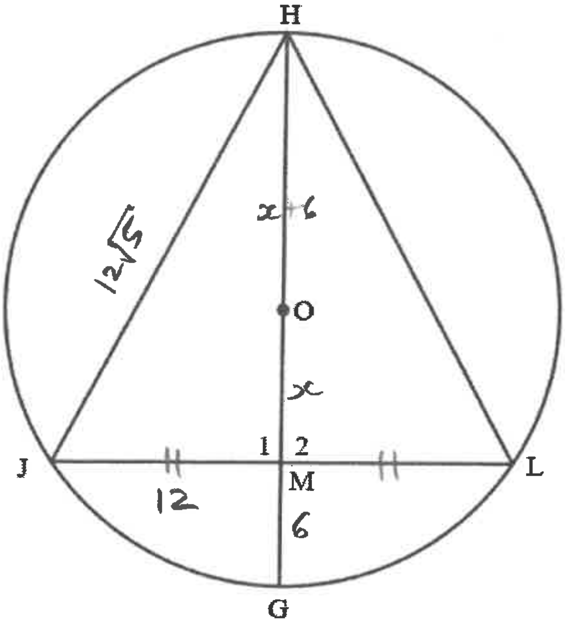
6.2.




| | | |
|---|---|---|
| 6.2.1. | OH $= x+6$ ✓ S    radii | |
| | HM $= x+6+x$ | |
| | $= 2x+6$ ✓ | 2 |
| | | |
| | | |
| 6.2.2. | $\hat{M}_1 = \hat{M}_2 = 90°$ ✓ SR    line from centre ⊙ | |
| | to midpt chord | |
| | | |
| | $(12\sqrt{5})^2 = (12)^2 + (2x+6)^2$    Pythag ✓ SR | |
| | $720 = 144 + 4x^2 + 24x + 36$ | |
| | $0 = 4x^2 + 24x - 540$ | |
| | $0 = x^2 + 6x - 135$ ✓    $\div 4$ | |
| | $0 = (x-9)(x+15)$ ✓ | |
| | $\therefore\ x = 9$ or $-\cancel{15}$ | |
| | $\therefore\ r = 9+6$    reject | |
| | $= 15$ ✓ | 5 |

OR

$(x+6)^2 = (x)^2 + (12)^2$ ✓ SR Pythag

$x^2 + 12x + 36 = x^2 + 144$

$12x = 108$ ✓

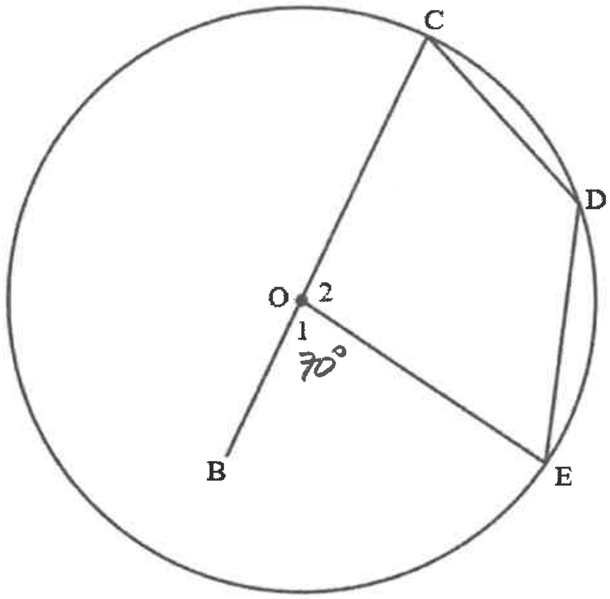
$x = 9$ ✓

$r = 9 + 6$

$= 15$ ✓

QUESTION 7

7.1.




| | | |
|---|---|---|
| | $C\hat{O}E$ (reflex) | |
| | $= 180° + 70°$      ^'s on str line $= 180°$ | |
| | $= 250°$ ✓SR | |
| | | |
| | $\hat{D} = 125°$ ✓S ✓R ^ @ centre = 2^ @ circumf | 3 |
| | | |
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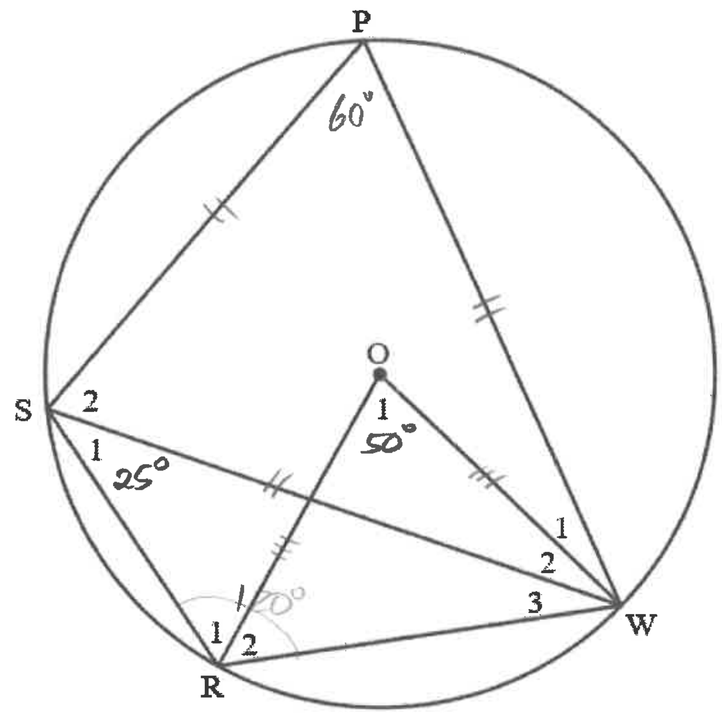
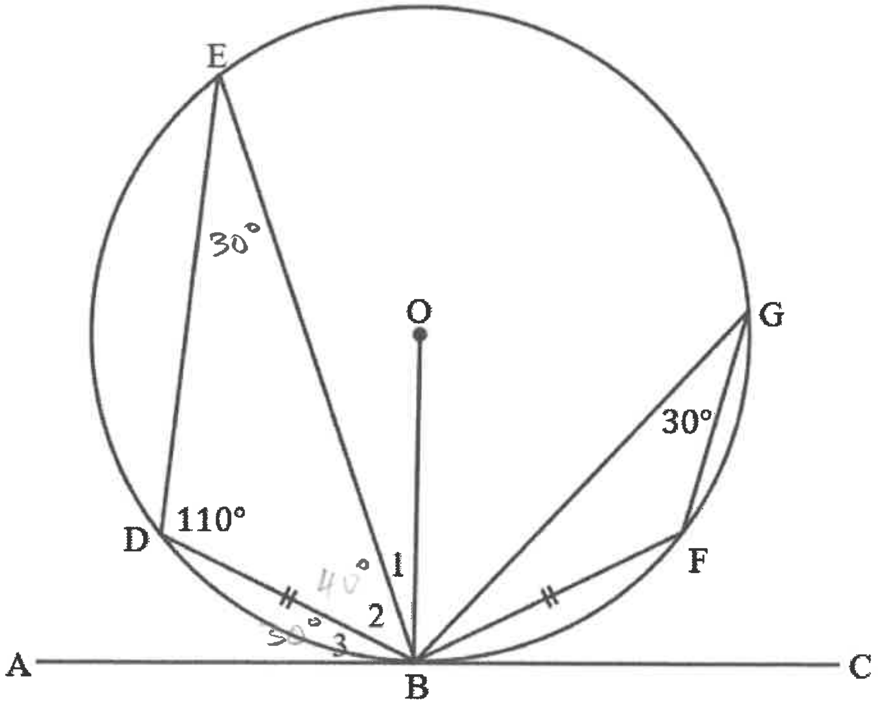
7.2.




| | | |
|---|---|---|
| 7.2.1 | $\hat{O}_1 = 50°$ ✓S ✓R ^@ centre = 2 × ^@ circumf | 2 |
| | | |
| | | |
| 7.2.2. | $\hat{P} = 60°$ ✓S ✓R equilat Δ | |
| | ^'s opp = sides | |
| | sum ^'s in Δ = 180° | |
| | $\therefore \hat{R}_1 + \hat{R}_2 = 120°$ ✓SR opp ^'s cyclic quad = 180° | |
| | | |
| | OR = OW radii | |
| | $\hat{R}_2 = \hat{W}_2 + \hat{W}_3$ ✓SR ^'s opp = sides | |
| | $\therefore \hat{R}_2 = \frac{180° - 50°}{2}$ sum ^'s in Δ = 180° | |
| | $= 65°$ ✓SR | |
| | | |
| | $\therefore \hat{R}_1 = 55°$ ✓ | 6 |
| | | |

7.3.




| | | |
|---|---|---|
| 7.3.1. | $\hat{E} = 30°$ ✓S ✓R = chords = ∠s @ circumf | |
| | $\hat{B}_3 = 30°$ ✓S ✓R tan chord thm | 4 |
| | | |
| | | |
| 7.3.2. | $\hat{B}_2 = 40°$ ✓SR sum ∠s in $\Delta = 180°$ | |
| | $\hat{B}_1 = 20°$ ✓S ✓R tan ⊥ rad | 3 |
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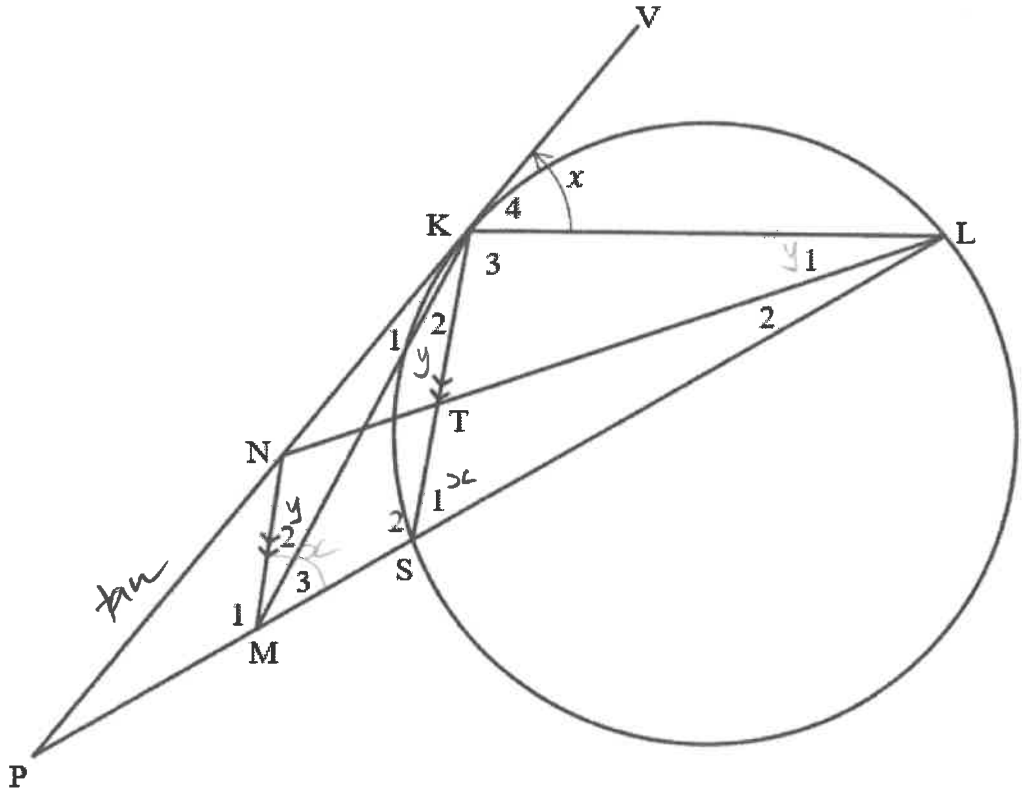
QUESTION 8

| | | |
|---|---|---|
| 8.1. | alternate circle segment | 2 |
| | | |

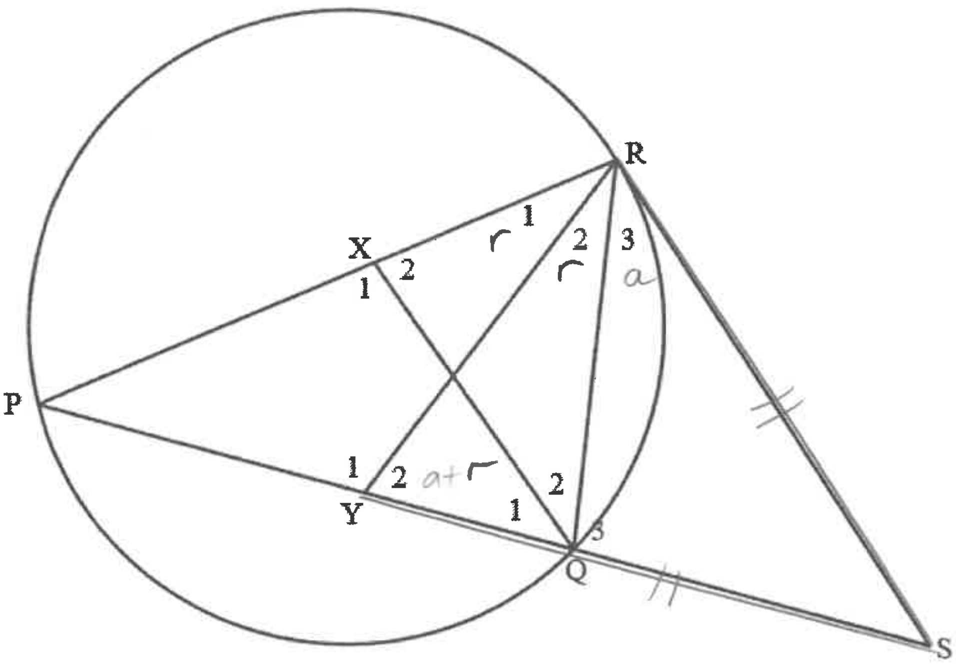
8.2.




| | | | |
|---|---|---|---|
| 8.2.1. | $\hat{S}_1 = x$ ✓S ✓R | tan chord thm | |
| | $\hat{M}_2 + \hat{M}_3 = x$ ✓SR | corr ∠'s =, MN ∥ SK | |
| | $\therefore \hat{K}_4 = \hat{M}_2 + \hat{M}_3$ ✓S | both $= x$ | |
| | ∴ KLMN is a ✓R | conv ext ∠ = | |
| | cyclic quad | cyclic quad | 5 |
| | | | |
| | | | |
| 8.2.2. | let $\hat{L}_1 = y$ | | |
| | $\therefore \hat{M}_2 = y$ ✓S ✓R | ∠'s in same ⊙ segm = | |
| | $\therefore \hat{K}_2 = y$ ✓SR | alt ∠'s =, MN ∥ SK | |
| | $\therefore \hat{L}_1 = \hat{K}_2$ | both $= y$ | 3 |
| | | | |
| | | | |

QUESTION 9

9.




| | | |
|---|---|---|
| | let $\hat{R}_1 = \hat{R}_2 = r$    given | |
| | $\hat{R}_3 = a$ | |
| | | |
| | $\hat{Y}_2 = a + r$ ✓SR   ∠'s opp = sides | |
| | $\hat{P} + r = a + r$   ext ∠ Δ | |
| | ∴ $\hat{P} = a$ ✓SR | |
| | | |
| | ∴ $\hat{R}_3 = \hat{P}$ ✓S   both = a | |
| | | |
| | ∴ SR is a tan ✓R   conv tan chord | |
| | to ⊙ at R   thm | 4 |
| | | |
| | | |
| | | |

## ADDITIONAL SPACE

## ADDITIONAL SPACE